MATH 2B/5B Prep: Product & Quotient Rule

1. Find the derivative of $f(x) = x^2 \sec(x)$.

Solution: Here we have a product of functions $g(x) = x^2$ and $h(x) = \sec(x)$, with derivatives

$$g'(x) = 2x \qquad h'(x) = \sec(x)\tan(x)$$

Then by product rule we get

$$f'(x) = \frac{\mathrm{d}}{\mathrm{d}x} g(x)h(x) = g'(x)h(x) + g(x)h'(x) = 2x\sec(x) + x^2\sec(x)\tan(x)$$

2. Compute the derivative of $\dfrac{\sin(3x)}{x}$.

Solution: This is a quotient of functions $f(x) = \sin(3x)$ and $g(x) = x$. They have derivatives

$$f'(x) = 3\cos(3x) \qquad g'(x) = 1$$

Note that $f(x)$ is a composition of functions and we had to use chain rule for its derivative. Then using quotient rule gives

$$\frac{\mathrm{d}}{\mathrm{d}x}\frac{f(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2} = \frac{x \cdot 3\cos(3x) - \sin(3x) \cdot 1}{x^2} = \frac{3x\cos(3x) - \sin(3x)}{x^2}$$

3. Find $\dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{\tan(x)e^x}{x^2}$

Solution: We have a quotient of functions $f(x) = \tan(x)e^x$ and $g(x) = x^2$. These have derivatives

$$f'(x) = \sec^2(x)e^x + \tan(x)e^x \qquad g'(x) = 2x$$

and note that $f'(x)$ uses product rule in finding the derivative. Then quotient rule says

$$\frac{\mathrm{d}}{\mathrm{d}x}\frac{\tan(x)e^x}{x^2} = \frac{\mathrm{d}}{\mathrm{d}x}\frac{f(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2} = \frac{x^2 \cdot (\sec^2(x)e^x + \tan(x)e^x) - \tan(x)e^x \cdot 2x}{x^4}$$